APSMO

2020 : DIVISION J

WEDNESDAY 25 MARCH 2020

MATHS OLYMPIAD

OLYMPIAD 1

Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

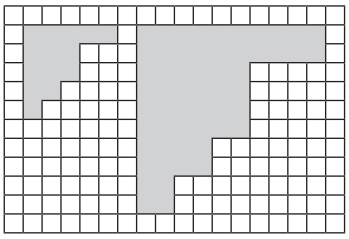
←

Keep your answers hidden by folding backwards on this line.

1A. Evaluate:

72 + 63 + 54 + 45 + 36 + 27.

1B. The smaller figure in the diagram has an area of 14 square units.



The larger figure on the right is produced by doubling the side lengths of the smaller figure.

What is the area of the larger figure, in square units?

1C. In a survey, 80 students were asked if they liked vanilla or chocolate cake.

Their responses were:

- 35 students liked vanilla,
- 32 students liked chocolate, and
- 24 students said they didn't like either vanilla or chocolate.

How many students said they liked both vanilla and chocolate?

1D. An *addy* number is a 5-digit number with the following properties:

- The first (leftmost) digit plus the second digit is the third digit.
- The second digit plus the third digit is the fourth digit.
- The third digit plus the fourth digit is the fifth (rightmost) digit.
- All of the digits are different.

How many different *addy* numbers are possible?

1E. Sophia has three stickers for every two stickers that Ethan has.

If Sophia gives Ethan 16 of her stickers, then Ethan will have twice as many stickers as Sophia has.

How many stickers do Sophia and Ethan have all together?

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MATHS OLYMPIAD

OLYMPIAD 1

1A.

Student Name:

1B.

1C.

1D.

1E.

Fold here. Keep your answers hidden.

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OLYMPIAD 1

Solutions and Answers

For teacher use only. Not for Distribution.

1A: 297	**1B:** 56	**1C:** 11	**1D:** 3	**1E:** 60

1A. **METHOD 1** ***Strategy:*** *Group the first and last, the second and the next-to-last, etc.*

72 + 63 + 54 + 45 + 36 + 27 = (72 + 27) + (63 + 36) + (54 + 45) = 99 + 99 + 99 = 3 × (100 – 1) = 300 – 3 = **297**.

METHOD 2 ***Strategy:*** *Consider the tens digits and the ones digits separately.*

72 + 63 + 54 + 45 + 36 + 27 = (70 + 60 + 50 + 40 + 30 + 20) + (2 + 3 + 4 + 5 + 6 + 7) = 270 + 27 = **297**.

Follow-Ups: *(1) Form a 3 × 3 magic square using the numbers 18, 27, 36, 45, 54, 63, 72, 81, and 90.*

63	72	27
18	54	90
81	36	45

(2) The numbers in the following sequence keep doubling: 1, 2, 4, 8, and so on.
If the numbers are added in the order 1 + 2 + 4 + 8 + …, how many numbers are needed in order to first surpass one million? [20]

1B. **METHOD 1** ***Strategy:*** *Use the two long sides of the figure to draw a rectangle.*

Create a rectangle and subtract the area that is not in the original figure.

The dimensions of the rectangle will be 10 units × 10 units, so it will have an area of 100 square units.

Then apply some method to determine the area that is not in the original figure - e.g. by counting the number of 1 × 1 squares or subdividing the unwanted portion into rectangles.

In either case, the unwanted region has an area of 44 square units.

The area of the figure on the right is 100 – 44 = **56** square units.

METHOD 2 ***Strategy:*** *Use the idea that doubling the length dimensions of a shape multiplies the area by 4.*

Multiply the area of the smaller figure on the left by 4 to find the area of the larger figure.

The larger figure has an area of 4 × 14 = **56** units^2.

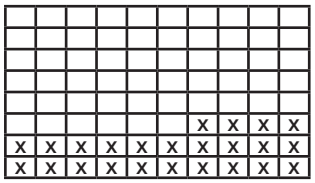
Follow-Ups: *(1) Find the area of a shape formed by halving each of the sides of the shape on the left.* [3.5 units^2]

(2) If the shape on the left is uniformly enlarged to create a similar shape that has an area of 224 square units, what is the perimeter of the resulting shape? [80 units]

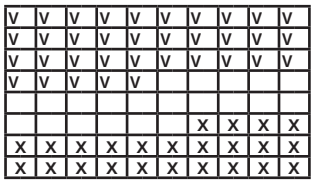
1C. **METHOD 1** ***Strategy:*** *Apply logic and reasoning.*

There are 80 students.

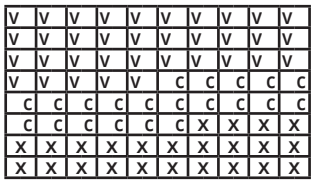
24 students do not like either flavour.



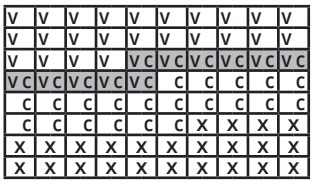
35 students like vanilla.



The remaining 80 – 24 – 35 = 21 students must only like chocolate.



Since 32 students like chocolate, 32 – 21 = **11** students must like both.

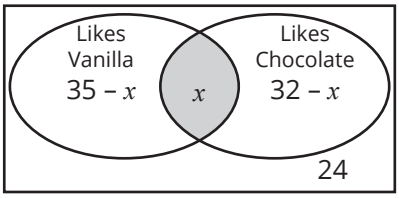


METHOD 2 ***Strategy:*** *Draw a Venn diagram and apply algebra.*

Let x indicate the number of students who like both vanilla and chocolate.

35 students like vanilla, so 35 – x only like vanilla (and not chocolate).

32 students like chocolate, so 32 – x only like chocolate (and not vanilla).



With 80 students in total,

$(35 - x) + x + (32 - x) + 24 = 80$

$35 - x + x + 32 - x + 24 = 80$

$91 - x = 80$, so $x =$ **11**.

There are **11** students who like both vanilla and chocolate.

***Follow-Up**: How many integers between 1 and 150 inclusive are NOT divisible by 2, 3, or 5?* [40]

1D. **METHOD 1** ***Strategy:*** *Make an organised list.*

Each digit after the first two digits is the sum of the two previous digits.

Since no digit can be repeated, 0 cannot be either the first or second digit since the sum will repeat a digit.

1st	2nd	3rd	4th	5th	*addy* number
1	2	3	5	8	12358
1	3	4	7	11	This does not work, as 11 cannot be a digit. So no other number beginning with 1 will work.
2	1	3	4	7	21347
2	3	5	8	13	This does not work, as 13 cannot be a digit.
3	1	4	5	9	31459

No other numbers satisfy the requirements. So there are **3** *addy* numbers.

METHOD 2 ***Strategy:*** *Apply algebra.*

Let the number be *ABCDE*. Then:

$A + B = C$ —①

$B + C = D$ —②

$C + D = E$ —③

Substitute ① into ②:

$B + (A + B) = D$

$A + 2B = D.$ —④

Substitute ① and ④ into ③:

$(A + B) + (A + 2B) = E$

$2A + 3B = E.$

Since E is a single digit, $2A + 3B \le 9$.

The only possible non-zero values for A and B are $(A, B) = (1, 2), (2, 1)$, and $(3, 1)$.

The **three** 5-digit numbers are 12358, 21347, and 31459.

***Follow-Up**: How many 3-digit positive numbers have the units digit as the product of the tens digit and the hundreds digit if:*

(1) all three digits are distinct? [4: 236, 326, 248, 428]

(2) no digit is zero and at least two digits are the same? [19: 111, 122, ..., 199, 212, 313, ..., 919, 224, 339]

(3) at least two of the digits are the same? [28: as for part (2), plus 100, 200, ..., 900]

1E. **METHOD 1** ***Strategy:*** *Create a table to determine a pattern.*

Before giving any stickers to Ethan, Sophia must have a number of stickers that is:

- a multiple of 3, and
- greater than 16.

Start the table with 18 stickers for Sophia.

Before		After		Final Ratio	Verdict
Sophia	Ethan	Sophia	Ethan		
18	12	2	28	1:14	no good
21	14	5	30	1:6	closer
...	...	...	...	...	...
36	24	20	40	1:2	bingo!

Therefore, Sophia and Ethan started with 36 + 24 = **60** stickers all together.

METHOD 2 ***Strategy:*** *Apply algebra.*

Suppose Sophia began with $3N$ stickers.

Ethan began with $2N$ stickers.

All together they had $5N$ stickers.

After the transfer, Ethan has $2N + 16$, Sophia has $3N - 16$.

Ethan now has twice as many as Sophia.

So: $2 \times (3N - 16) = 2N + 16$

$6N - 32 = 2N + 16$

$4N = 48$

$N = 12.$

Therefore, Sophia and Ethan began with $5 \times 12 =$ **60** stickers.

***Follow-Up**: Emma has three stickers for every four stickers that Steven has.*

Together they have more than 125 stickers, but less than 130 stickers.

How many stickers must Steven give Emma so that they each have the same number of stickers? [9]